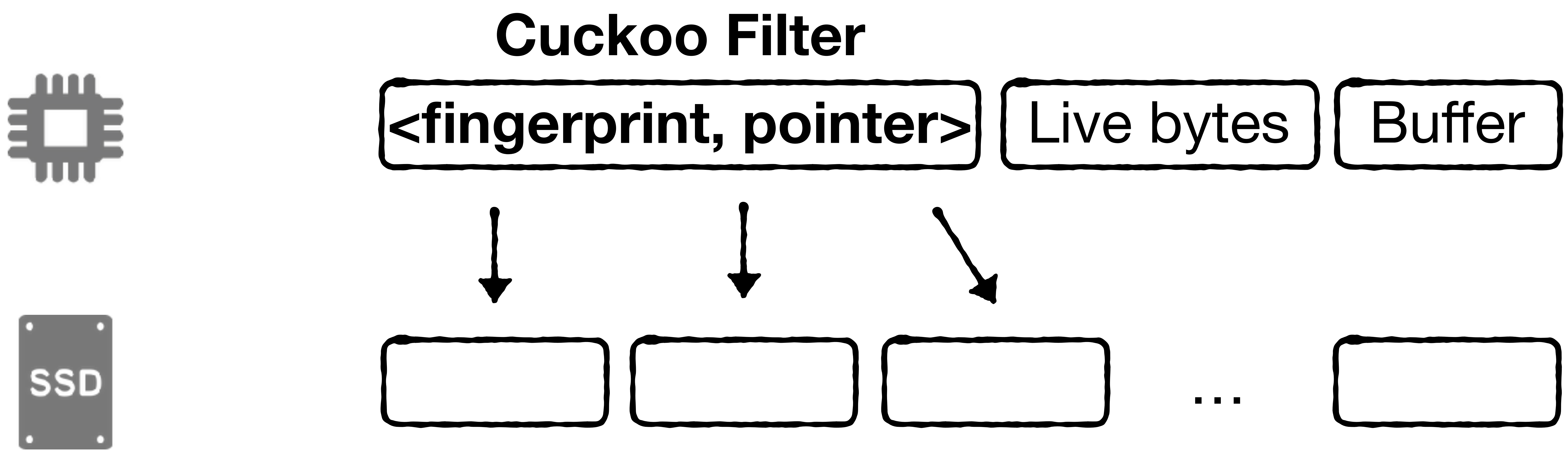
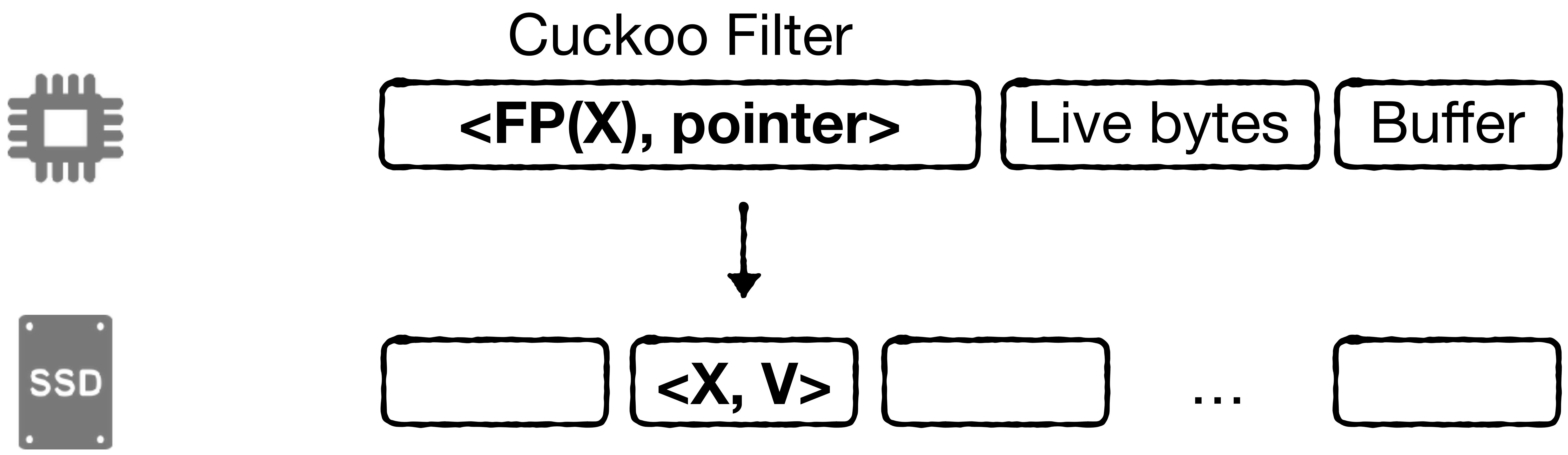
Tutorial on Circular Logs & Cuckoo Filters

Database System Technology

Niv Dayan

	Cuckoo Filter	Bloom filter
false positive rate:	$\approx 2^{-M+3}$	$2^{-M \cdot \ln(2)}$
Memory accesses for positive query	1.5	$M \cdot \ln(2)$
Memory accesses for negative query	2	2
Insertion cost	O(1)	$M \cdot \ln(2)$
Deletes?	**1.5**	**N/A**
Storing Payloads?	**Yes**	**N/A**


Cuckoo Filter
<fingerprint, pointer>
Live bytes
Buffer
SSD
...


Cuckoo Filter
<FP(X), pointer>
Live bytes
Buffer
SSD
<X, V>
...

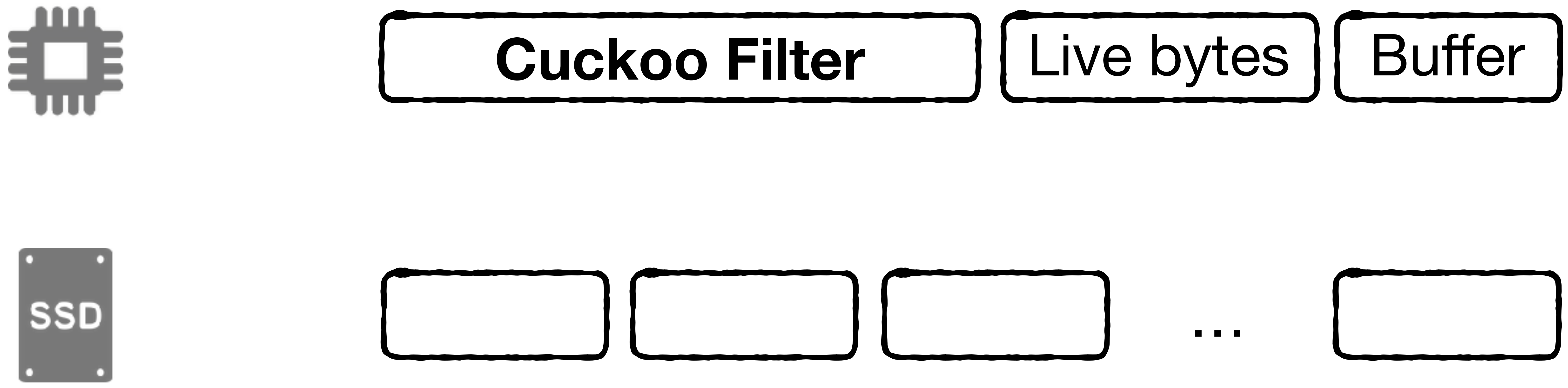
Index size = N * (P + **K**) * α

N = data size

P = pointer size = $O(\log_2 N/B)$

Our goal was reducing this → **K = key size = $\Omega(\log_2 N)$**

α = collision resolution overheads ≈0.8 → ≈0.95



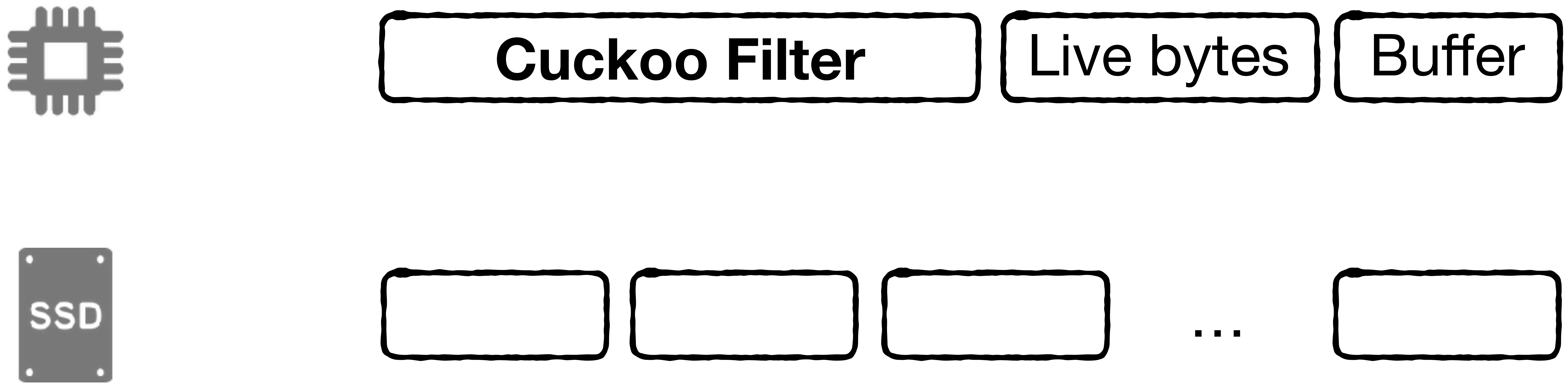
Index size = $N * (P + \mathbf{M}) / \alpha$

N = data size

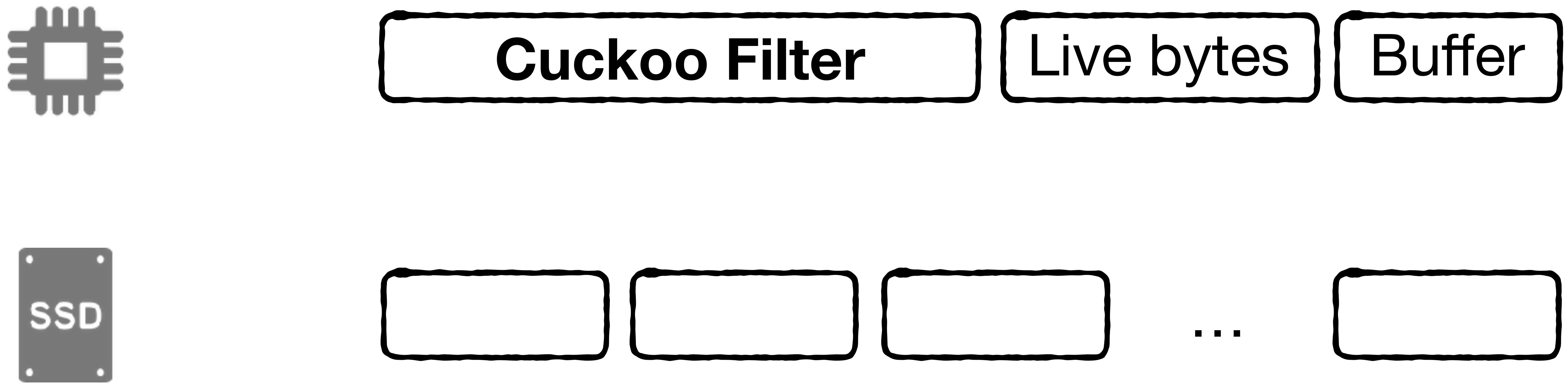
P = pointer size = $O(\log_2 N/B)$

Our goal was reducing this → **K = key size = $\Omega(\log_2 N)$** → **M bits / entry**

α = collision resolution overheads $\approx 0.8 \rightarrow \approx 0.95$

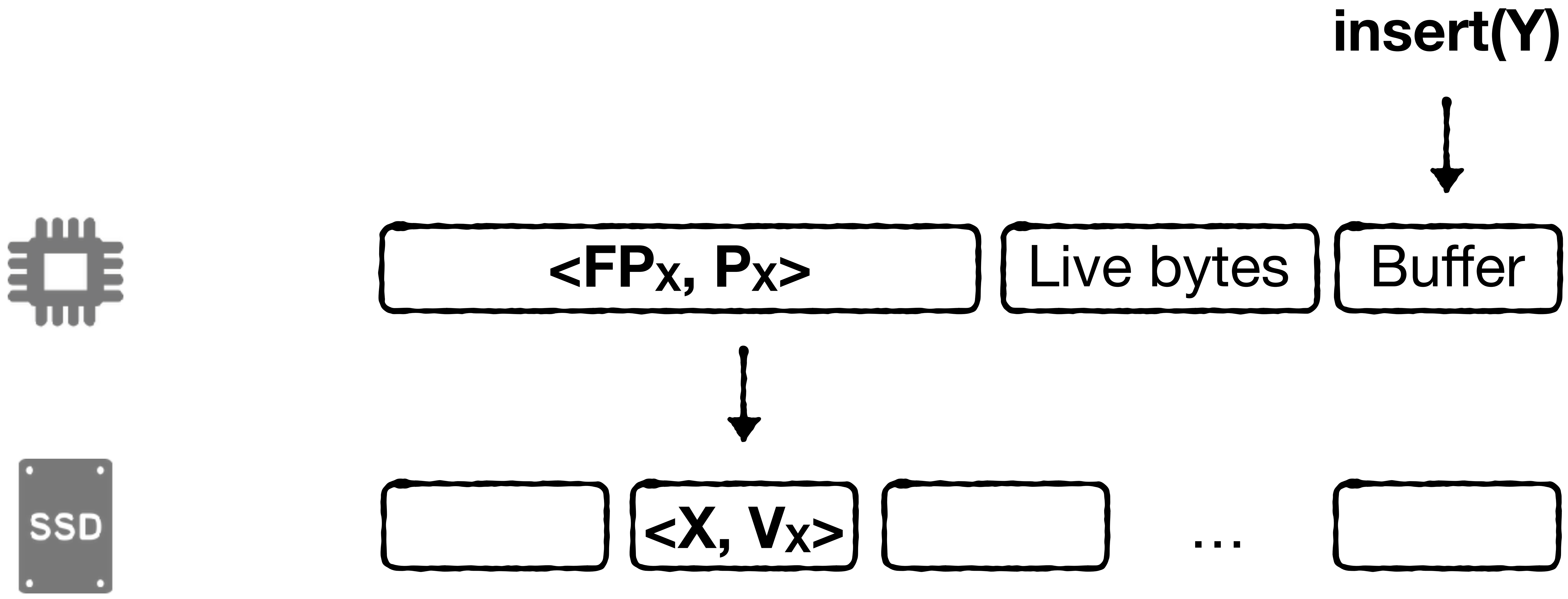


Implication of Using a Filter on Insertions/Deletes/Updates



Implication of Using a Filter on Insertions/Deletes/Updates

Suppose we insert key Y that has a matching fingerprint to existing key X ($FP_X = FP_y$)



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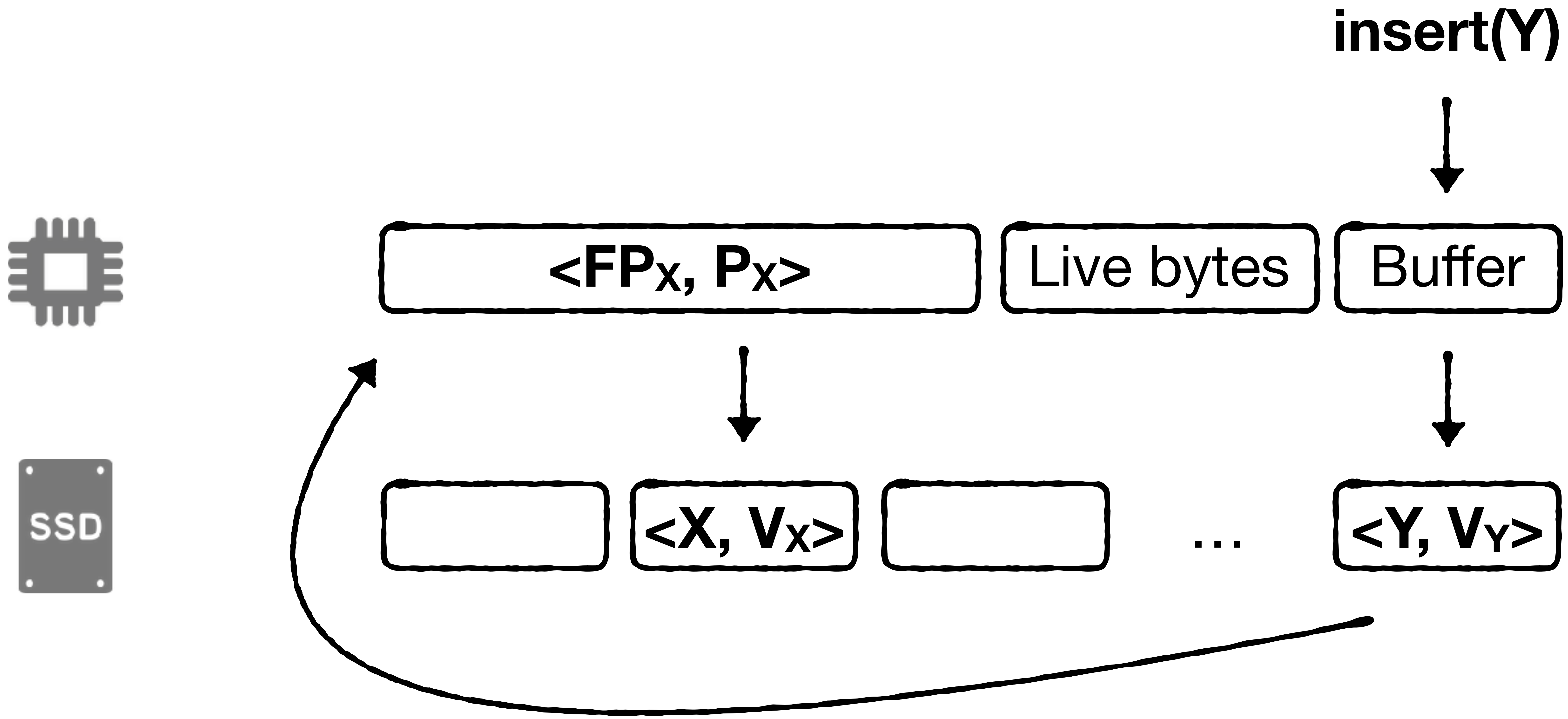

Need to set up pointer to Y

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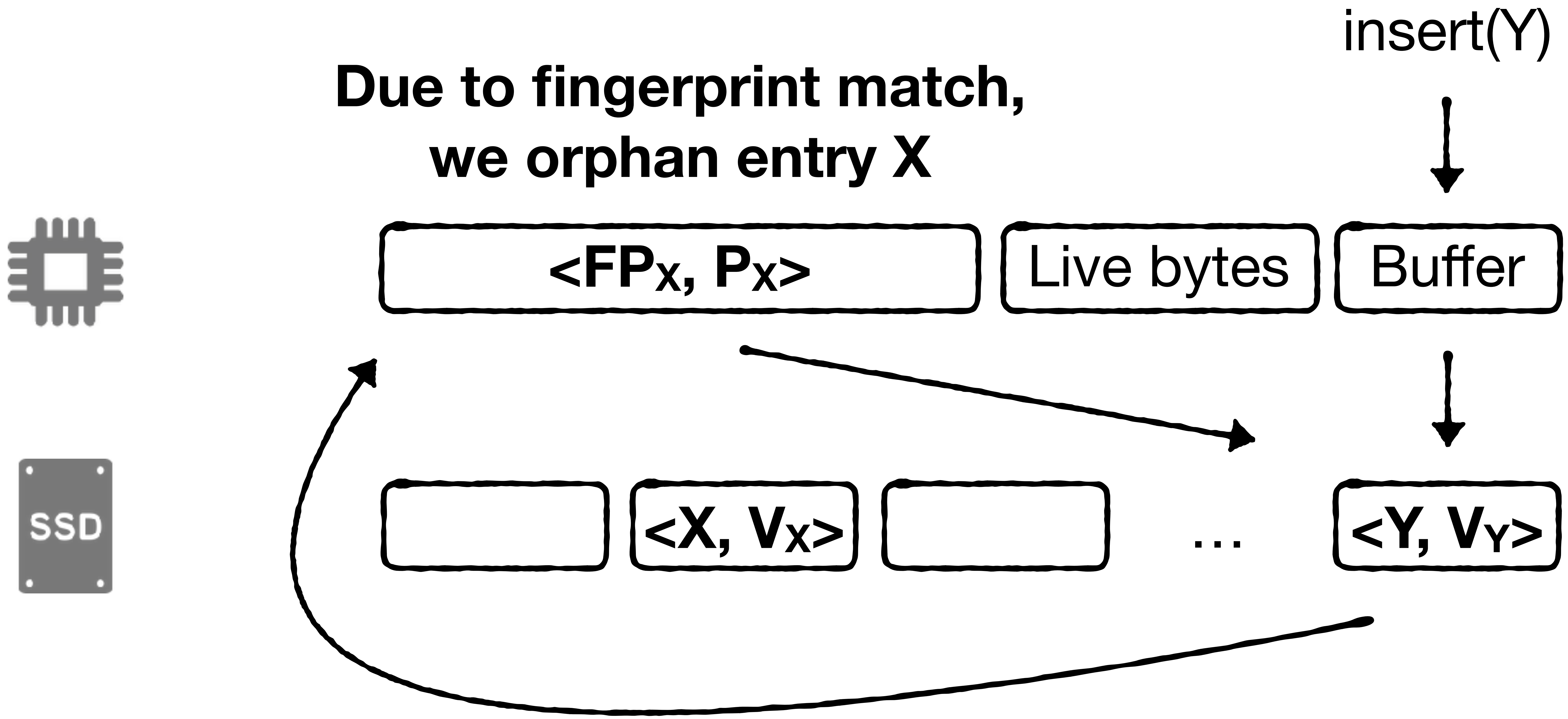

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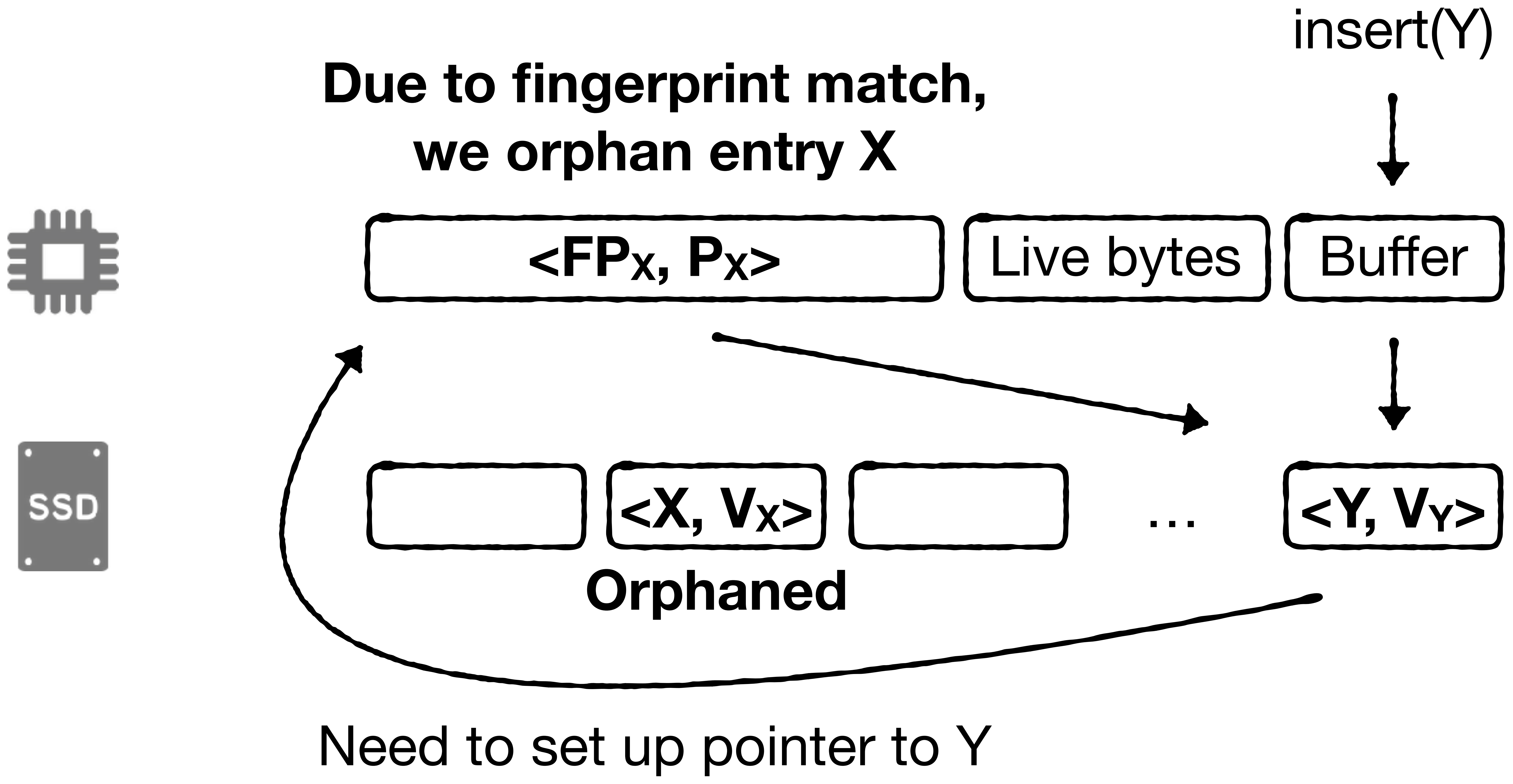

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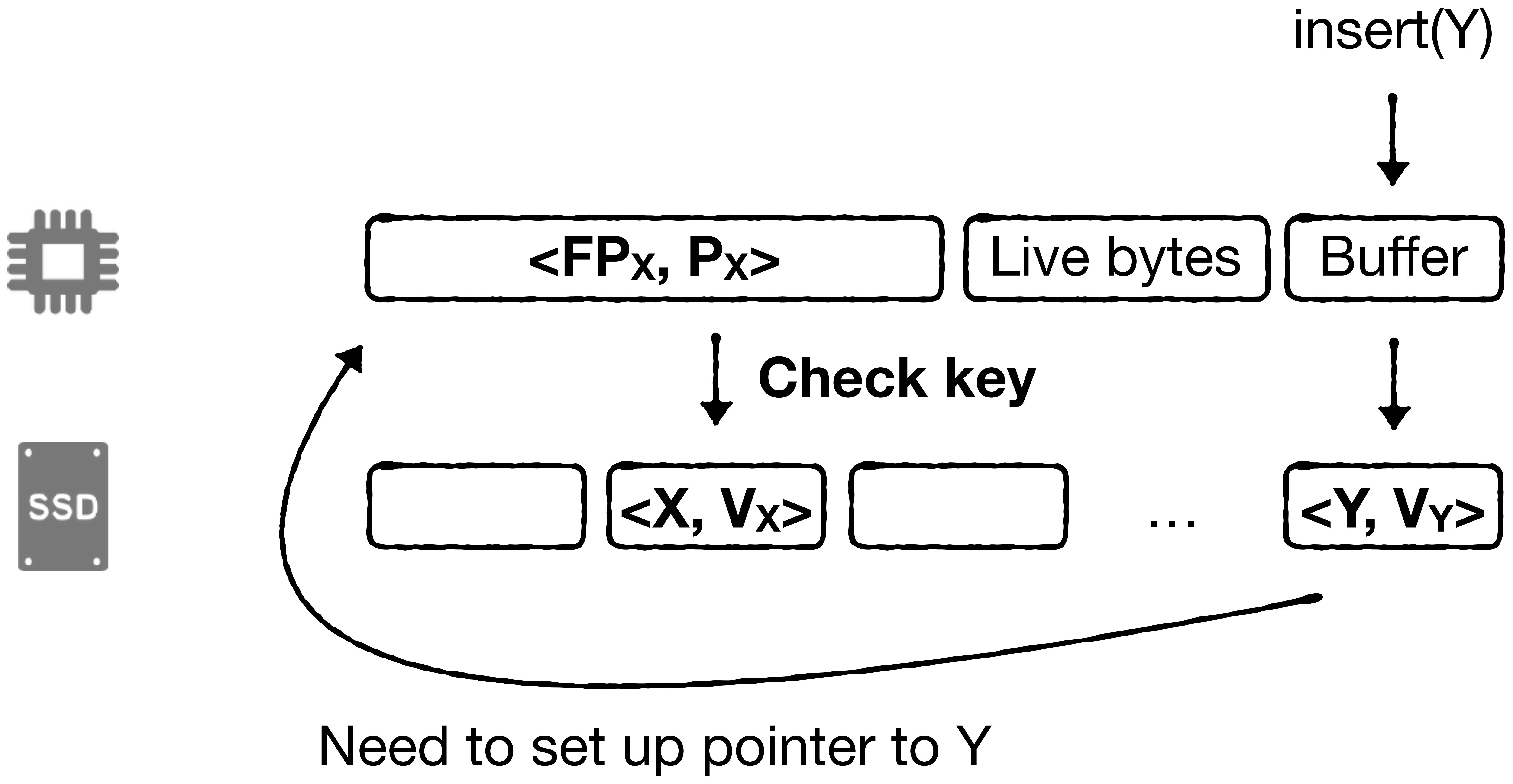
Suppose we insert key Y that has a matching fingerprint to existing key X

To safeguard against orphaning, we must issue read-before-write



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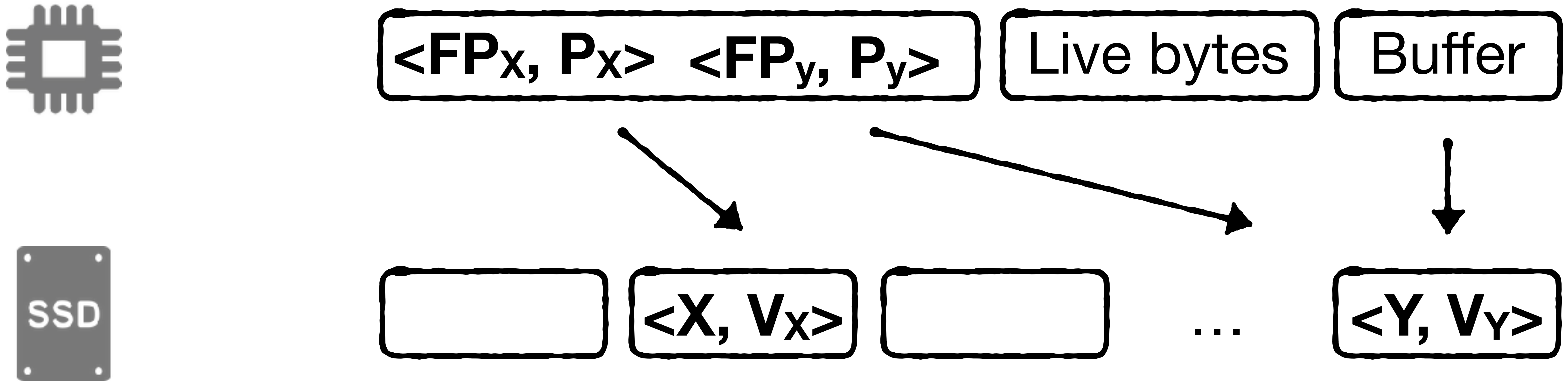
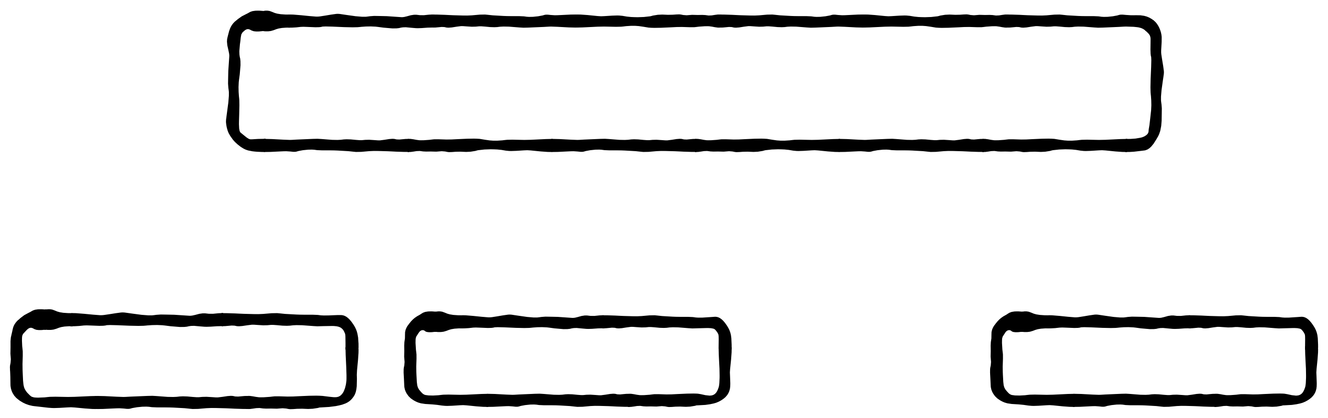


Circular Log w.
Cuckoo Filter



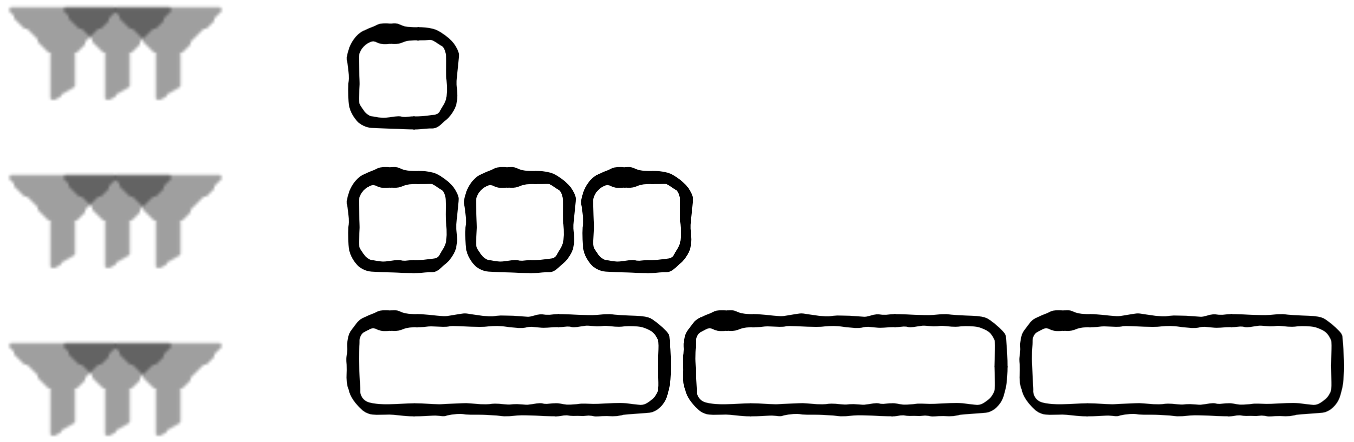
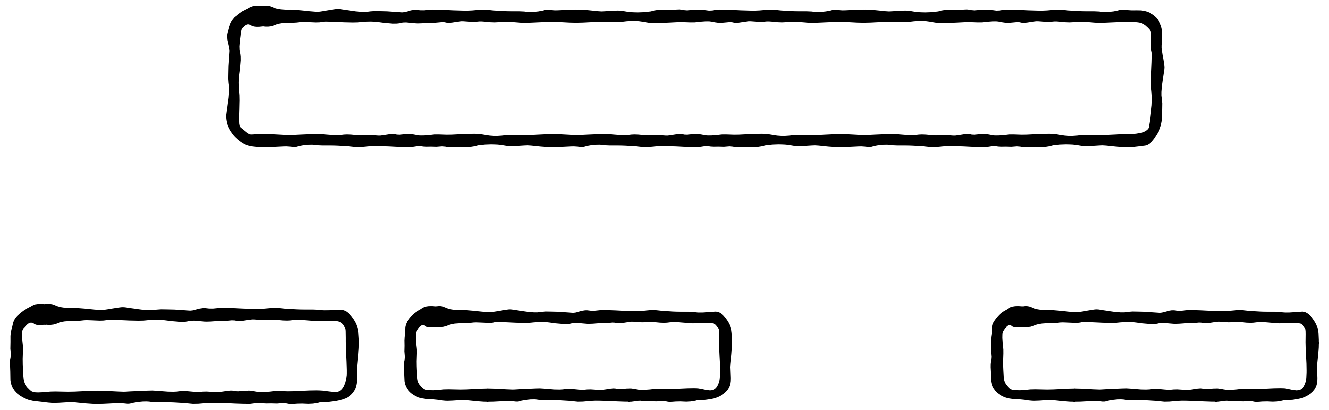
Updates/
Deletes: $O(1+2^{-M+3})$ reads & $O(GC/B)$ writes (excluding checkpointing)

Insert: $O(2^{-M+3})$ read & $O(GC/B)$ writes

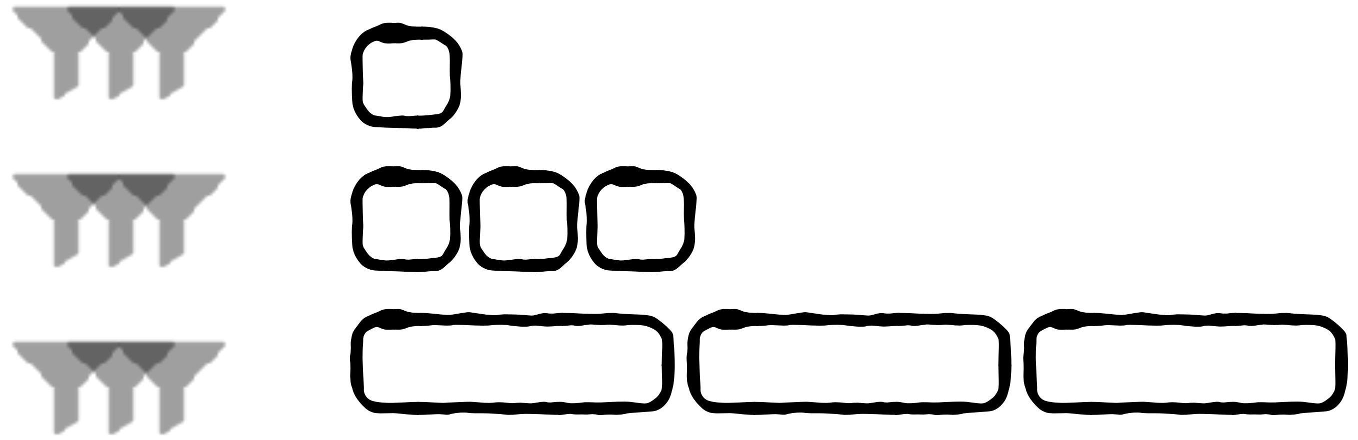
Gets: $O(1+2^{-M+3})$

Scan: $O(N/B)$

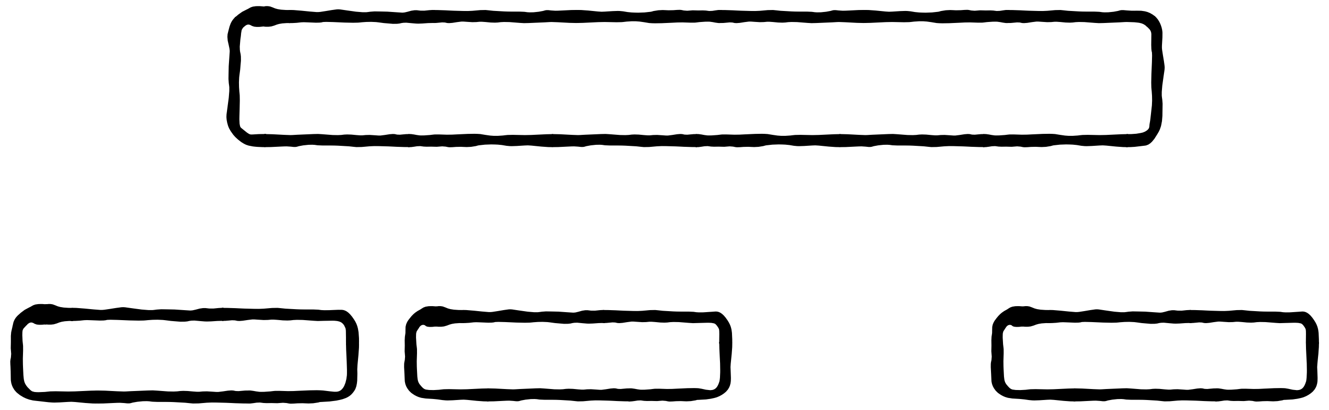
Memory
(bits / entry) $O(\log_2(N/B) + M))$

	Basic LSM-tree w. Monkey	Circular Log w. Cuckoo Filter
		
Updates/ Deletes:	$O(\log_2(N/P) / B)$ reads & writes	$O(1+2^{-M+3})$ reads & $O(GC/B)$ writes
Insert:	$O(\log_2(N/P) / B)$ reads & writes	$O(2^{-M+3})$ read & $O(GC/B)$ writes
Gets:	$O(1+2^{-M * \ln(2)})$	$O(1+2^{-M+3})$
Scan:	$O(\log_2 N/P + S/B)$	$O(N/B)$
Memory (bits / entry)	$O(K/B + M)$	$O(\log_2(N/B) + M))$

Basic LSM-tree w. Monkey



Circular Log w. Cuckoo Filter



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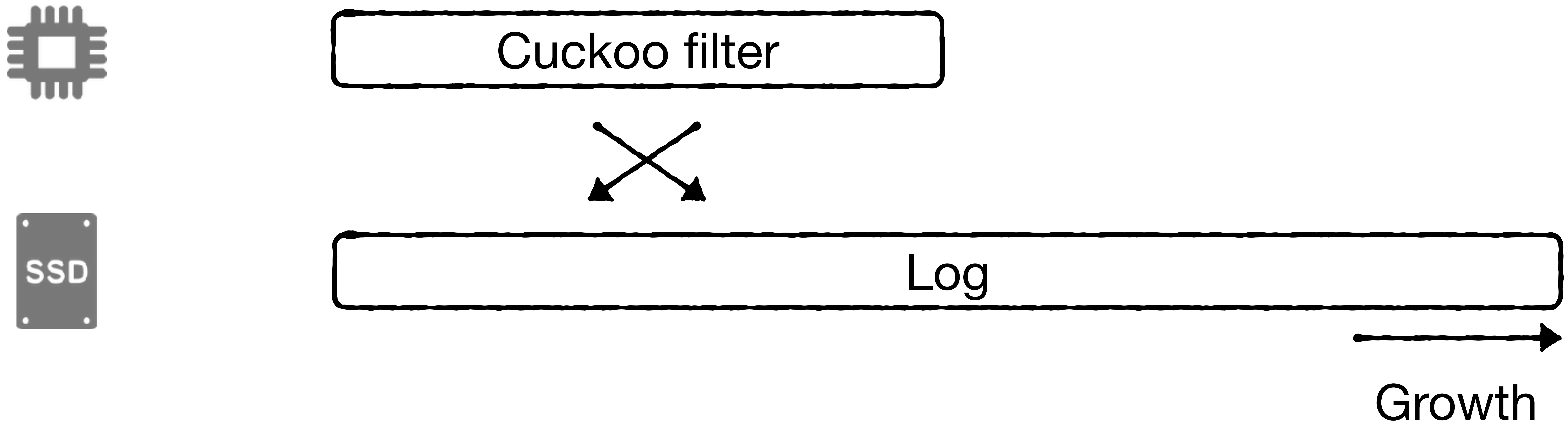
Scan:	$O(\log_2 N/P + S/B)$	$O(N/B)$
Memory (bits / entry)	$O(K/B + M)$	$O(\log_2(N/B) + M))$
Recovery	**Swift**	**Longer**

Question 1 - Expanding Cuckoo Filters

A Cuckoo filter is allocated with a fixed capacity. As a circular log grows, we must expand its cuckoo filter to map more data. Devise a Cuckoo filter expansion algorithm. Ideally, this algorithm should maintain constant time performance and not have to read any data from storage. Comment on any trade-offs or downsides.



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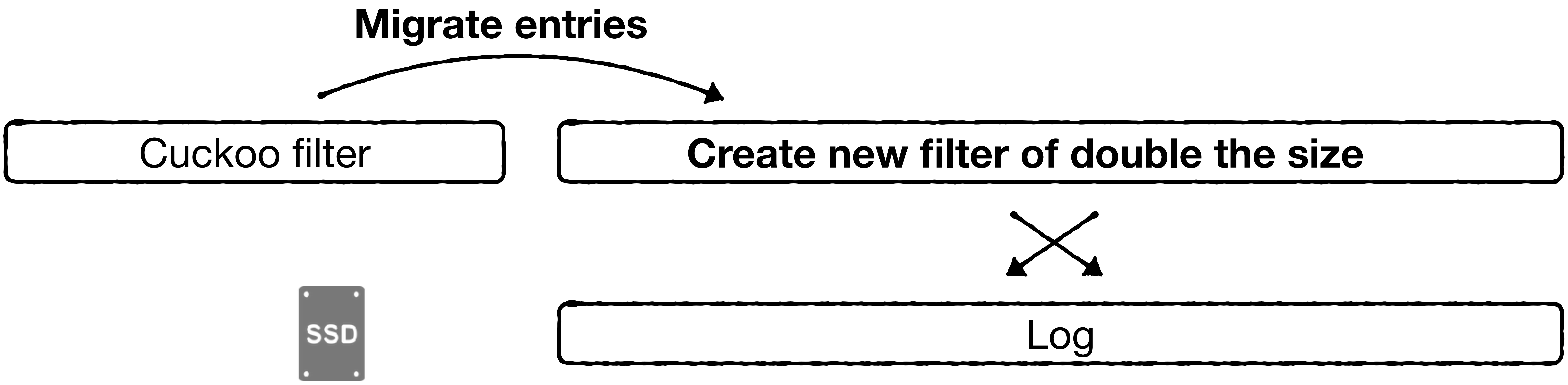

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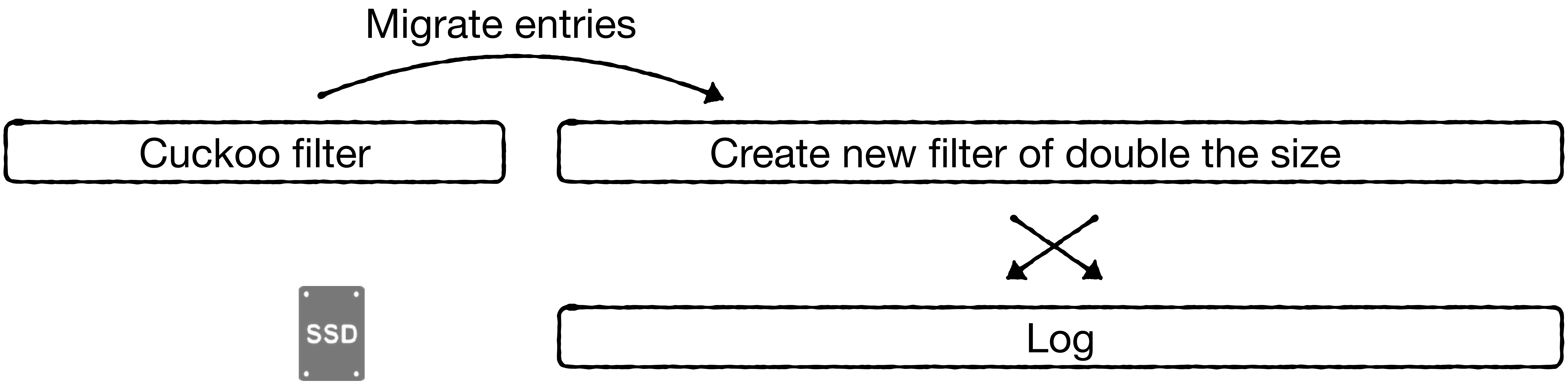
Challenge: we do not have the full keys to rehash. We only have fingerprints.



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Approach: view fingerprint and first bucket address as components of the same hash

address fingerprint

hash(X) = 01001 001110

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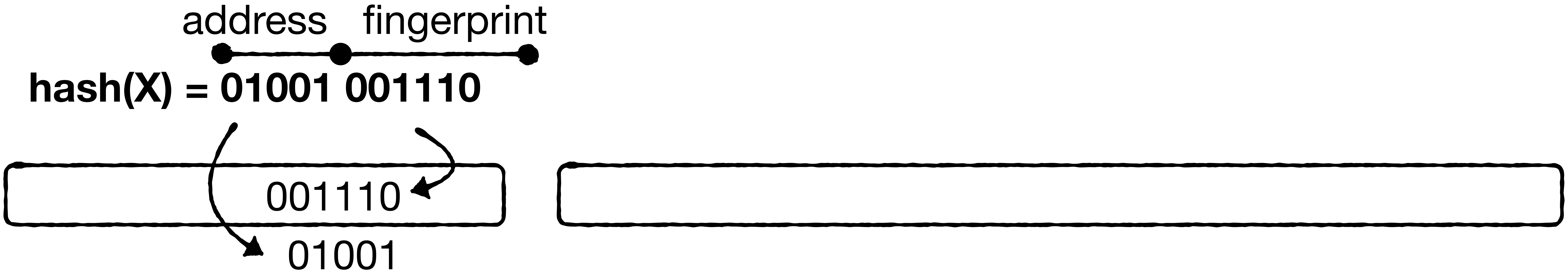

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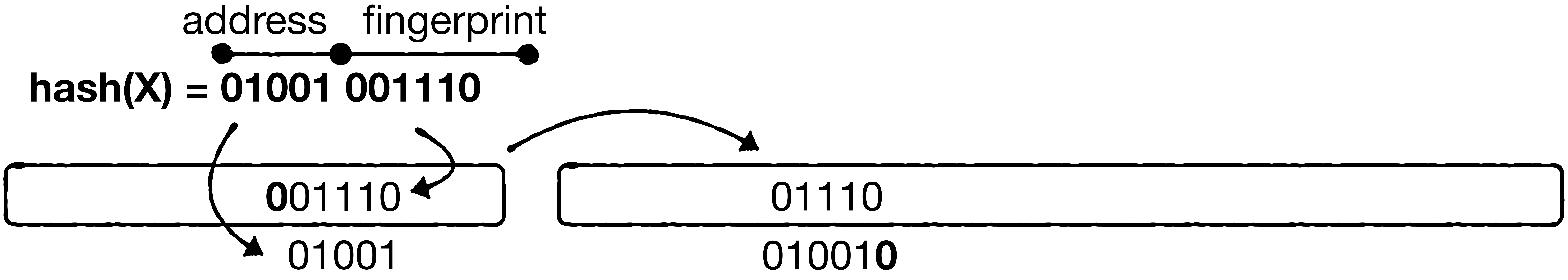
To migrate, transfer one bit from fingerprint to address



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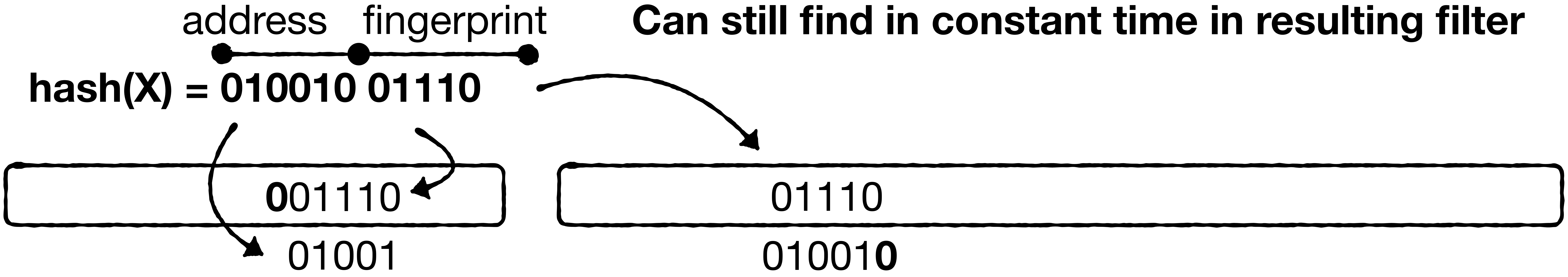
To migrate, transfer one bit from fingerprint to address

Can still find in constant time in resulting filter



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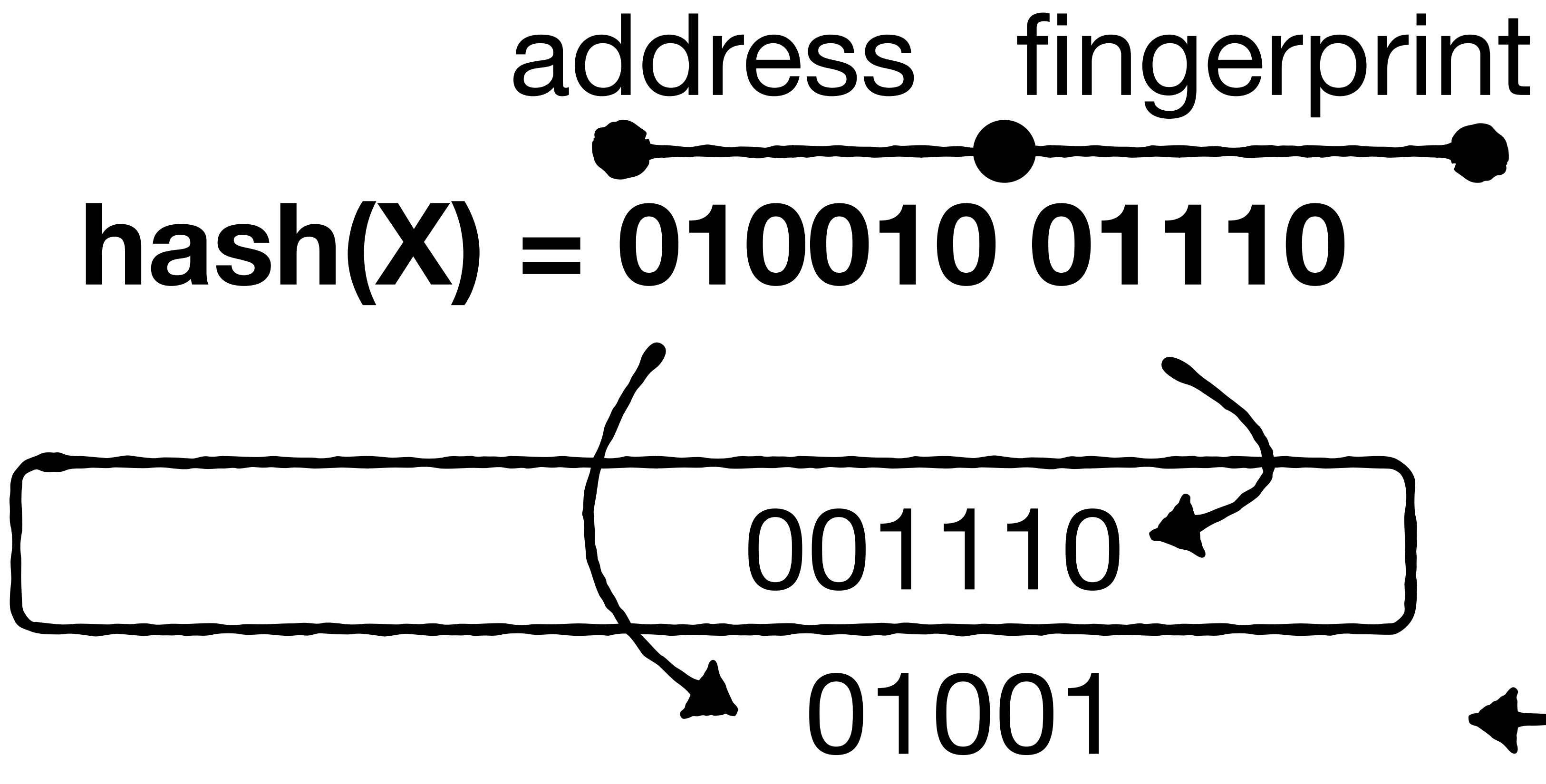
Complication: in a cuckoo filter an entry can be in one of two buckets, the canonical address and the alternative address. Only the canonical address should be viewed as a part of the original hash.



Alternative address = Canonical address XOR h(fingerprint)

Canonical or alternative address?

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Complication: in a cuckoo filter an entry can be in one of two buckets, the canonical address and the alternative address. Only the canonical address should be viewed as a part of the original hash.

Solution: add a bit to indicate whether the current address is canonical or alternative. If alternative, switch to canonical via XOR.

0 001110

01001

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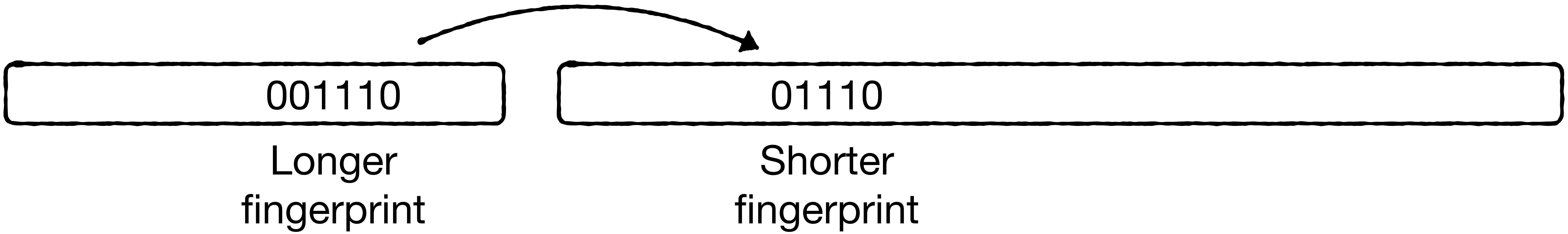
Every time we double the data size, we lose one bit from all fingerprints. Hence, the false positive rate increases: $O(2^{-M + 3 + \log(N)})$



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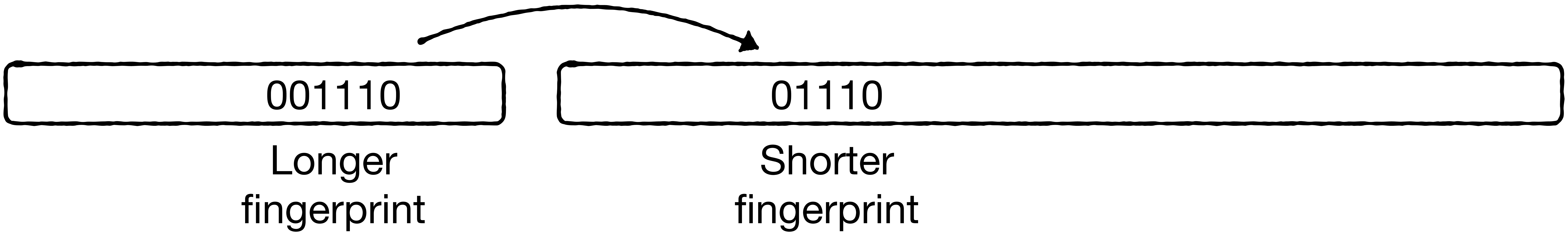
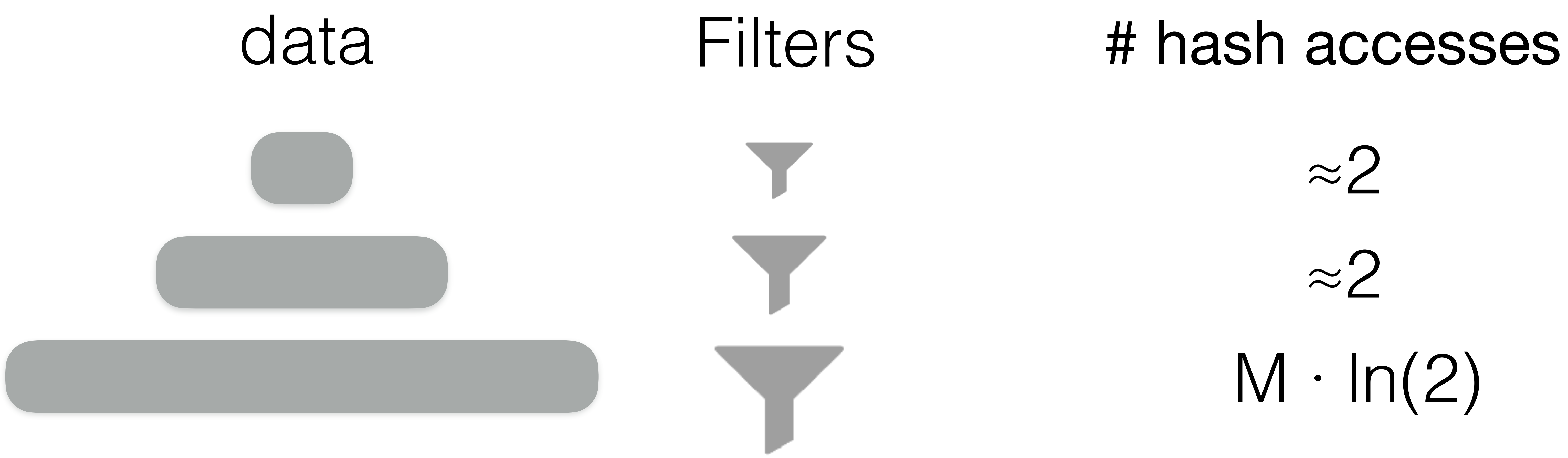


Question 2 - Cuckoo Filter for LSM-tree

As we have seen, the expected worst case query cost over Bloom filters for a basic LSM-tree is $O(L + M)$, where L is the number of levels and M is the number of bits per entry. (Assume only unique entries in the tree).

(A) How can we employ a cuckoo filter to achieve constant time?

(B) What are the implications on the false positive rate and memory footprint? Any downsides compared to plain Bloom filters?



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Unified Cuckoo filter

Filter accesses:	$O(1)$
False positive rate:	$O(2^{-M+3})$
Memory (bits/entry)	$O(M + \log_2(L))$
Construction:	$O(L)$

FP(X), 2
FP(Y), 3
…

<M bit Fingerprint, level>

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	Unified Cuckoo filter	**With Bloom filters**
Filter accesses:	$O(1)$	$O(M+L)$
False positive rate:	$O(2^{-M+3})$	$O(L \cdot 2^{-M \cdot \ln(2)})$
Memory (bits/entry)	$O(M + \log_2(L))$	$O(M)$
Construction:	$O(L)$	$O(L \cdot M)$

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	Unified Cuckoo filter	**With Bloom filters**	**Monkey**
Filter accesses:	$O(1)$	$O(M+L)$	$O(M+L)$
False positive rate:	$O(2^{-M+3})$	$O(L \cdot 2^{-M \cdot \ln(2)})$	$O(2^{-M \cdot \ln(2)})$
Memory (bits/entry)	$O(M + \log_2(L))$	$O(M)$	$O(M)$
Construction:	$O(L)$	$O(L \cdot M)$	$O(L \cdot (L + M))$

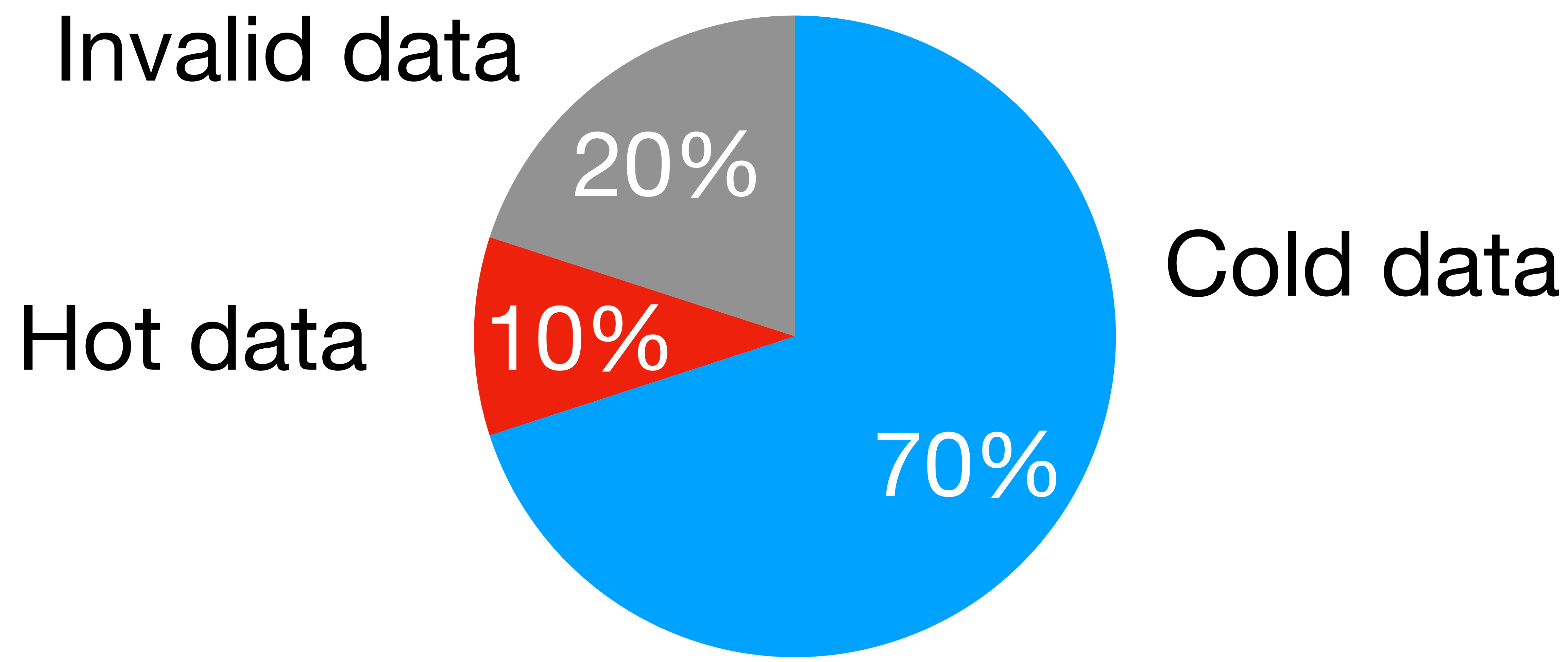
(This memory analysis here only account for the filters and not the fence pointers (internal nodes) being stored in memory)

Question 3 - Hot/Cold Data Separation

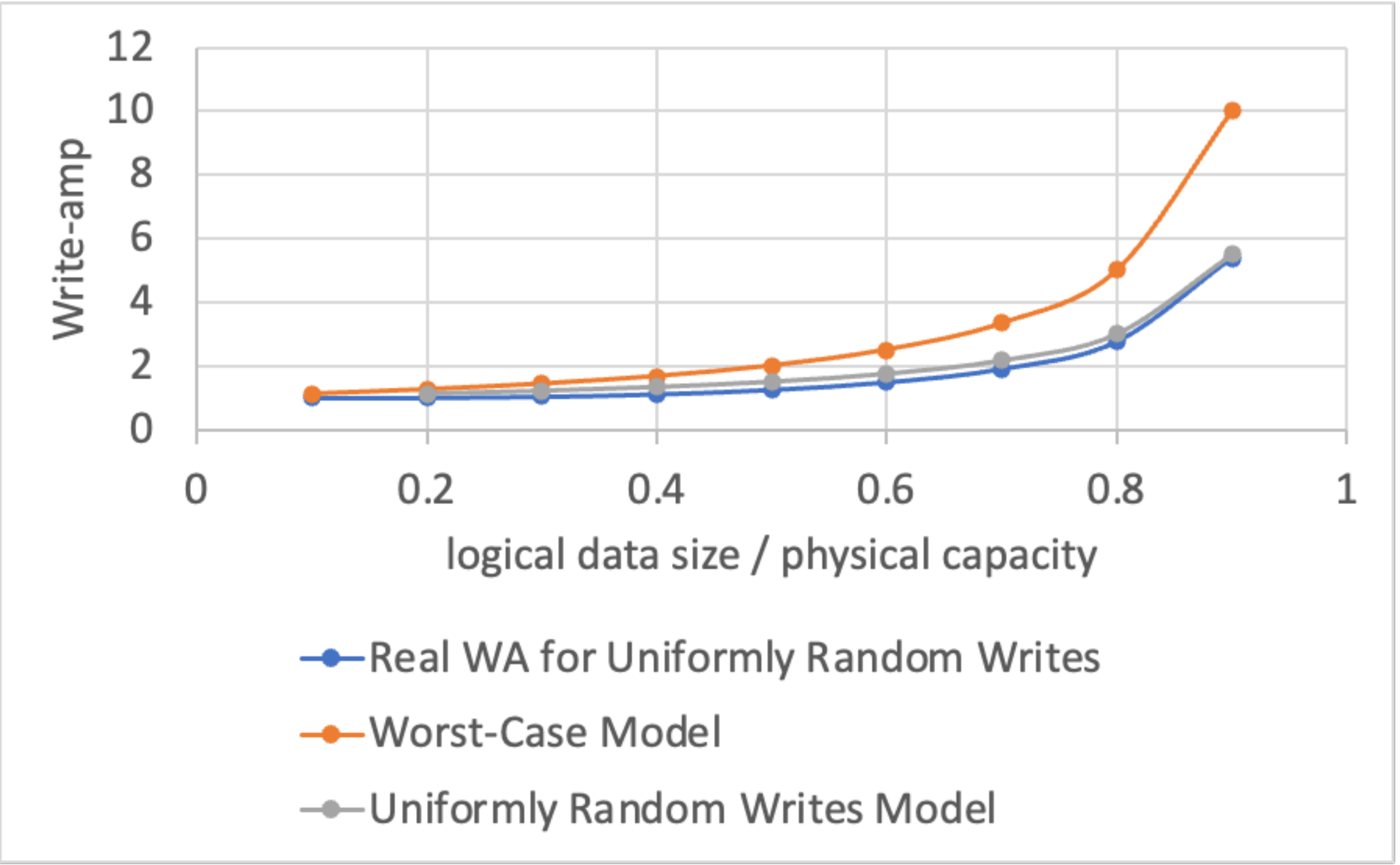
Consider a circular log where the physical capacity consists of 70% static data (never updated), 10% hot data, and 20% over-provisioning.

(A) Estimate a lower bound and upper bound for write-amplification assuming no hot/cold data separation.

(B) Estimate write-amplification assuming perfect hot/cold data separation.



Garbage-Collection Write-Amplification



$$1 + \frac{L/P}{1 - L/P}$$ **Worst case**

$$1 + \frac{1}{2} \cdot \frac{L/P}{1 - L/P}$$ **Uniformly random**

L = logical data size

P = physical data size

Garbage-Collection Write-Amplification

$$1 + \frac{L/P}{1 - L/P} = 1 + \frac{L}{P - L}$$ **Worst case**

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Garbage-Collection Write-Amplification

$$1 + \frac{L/P}{1 - L/P} = 1 + \frac{L}{P - L} = 1 + \frac{L}{O}$$ **Worst case**

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O = Overprovisioned space (P-L)

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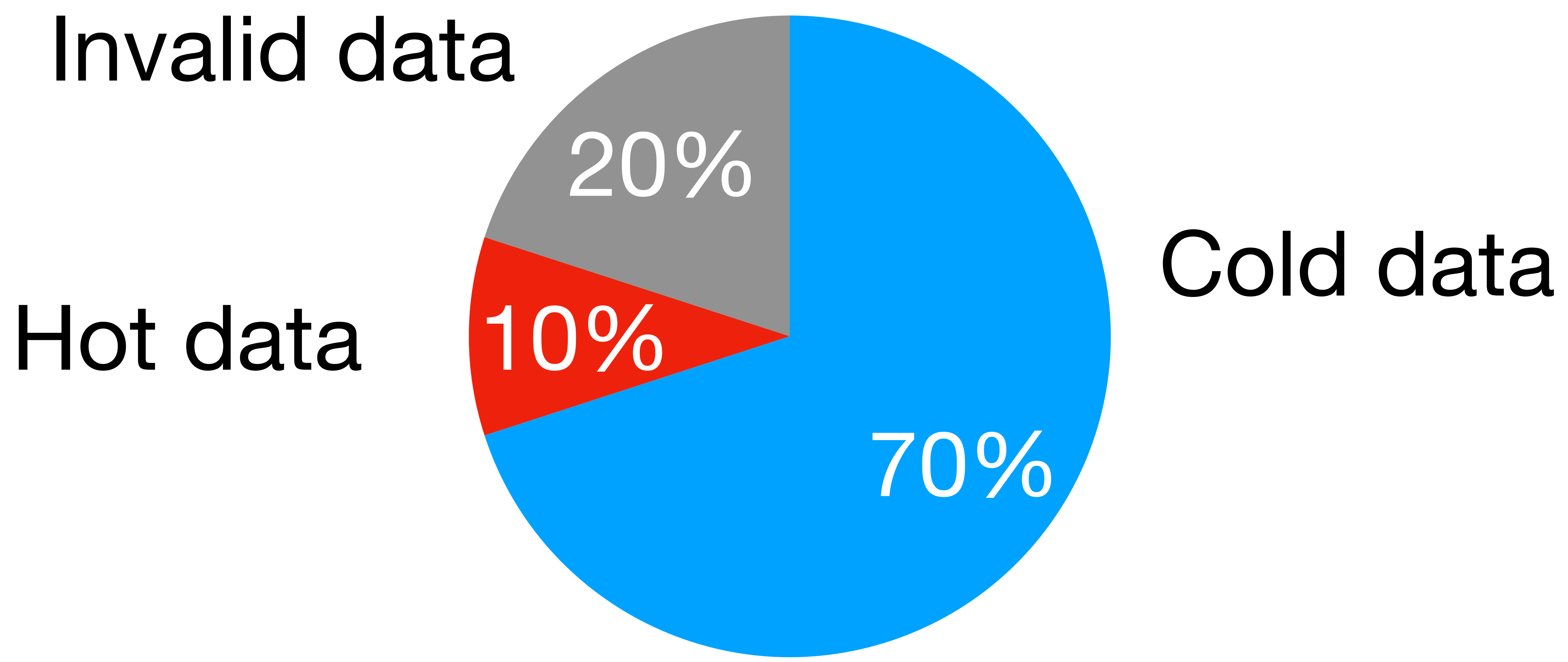

Worst case WA: $= 1 + \frac{L}{O}$

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Let C=0.7, H=0.1 and O=0.2

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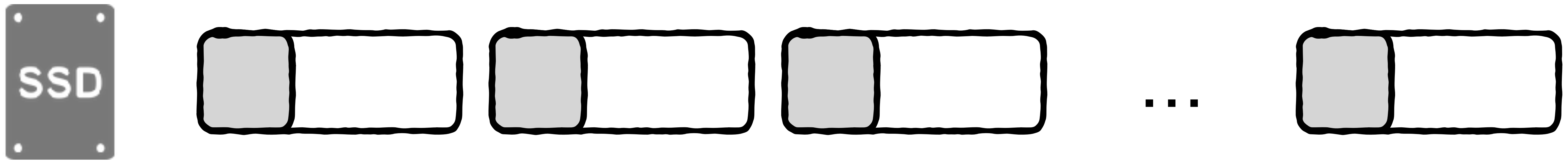
In worst-case, same amount of live data in each area



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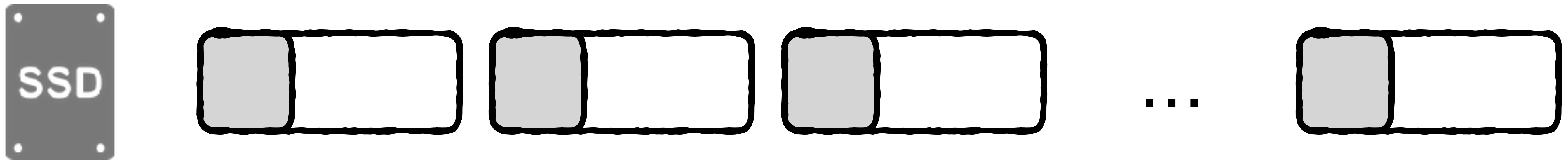

Upper bound: $= 1 + \frac{L}{O} = 1 + \frac{H + C}{O} = 1 + \frac{0.8}{0.2} = 5$

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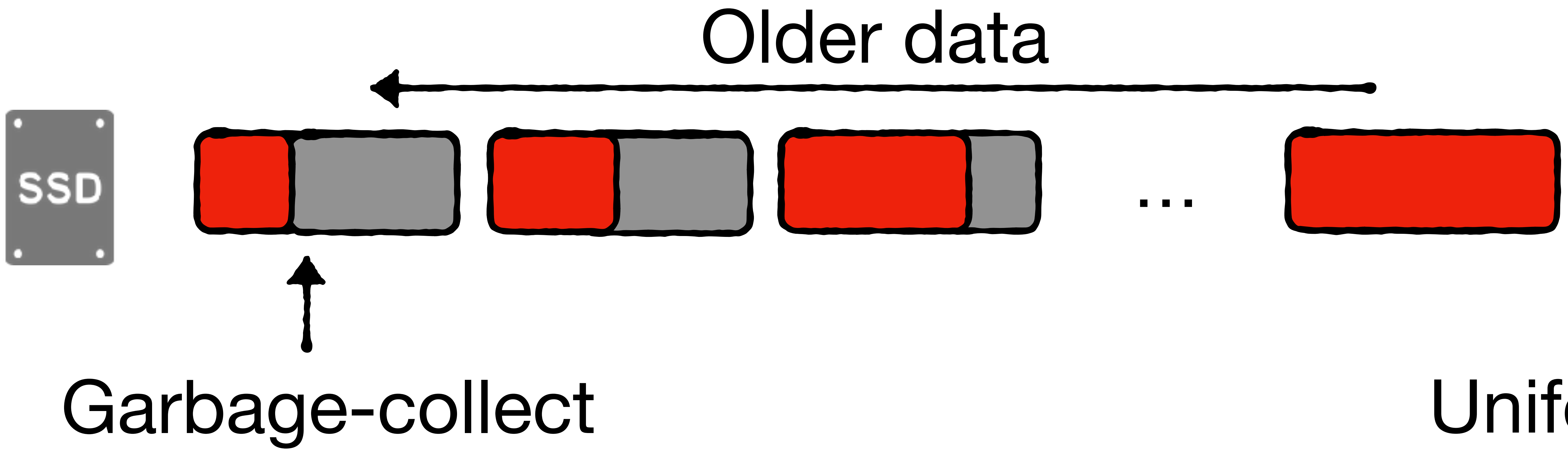
For lower bound, let's use our uniform workload distribution estimation.



Uniformly random: $= 1 + \frac{1}{2} \cdot \frac{L}{O}$

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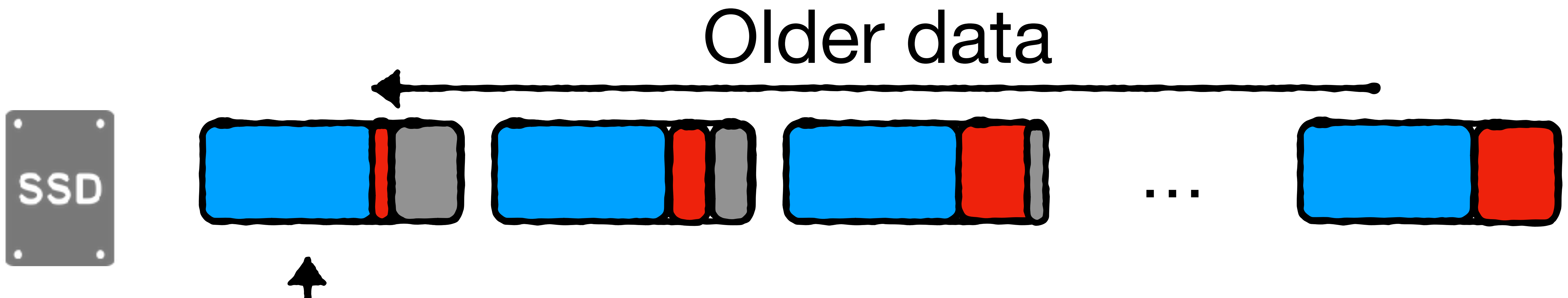


Garbage-collect

It works as a Lower bound since static data can only increase fraction of valid data in areas we garbage-collect

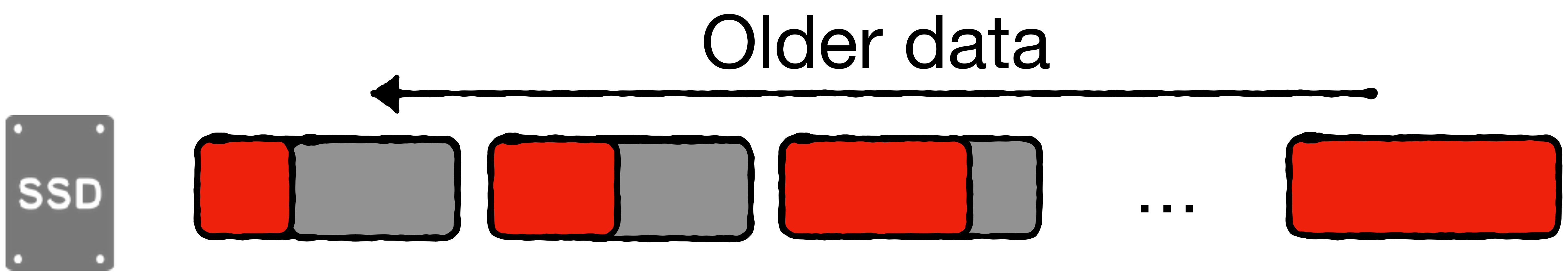
Question 3 - Hot/Cold Data Separation

Consider a circular log where the physical capacity consists of 70% static data (never updated), 10% hot data, and 20% over-provisioning.

(A) Estimate a lower bound and upper bound for write-amplification assuming no hot/cold data separation.

Let C=0.7, H=0.1 and O=0.2

For lower bound, let's use our uniform workload distribution estimation.



Lower bound: $= 1 + \frac{1}{2} \cdot \frac{L}{O} \quad = 1 + \frac{1}{2} \cdot \frac{H+C}{O} = 1 + \frac{1}{2} \cdot \frac{0.8}{0.2} = 3$

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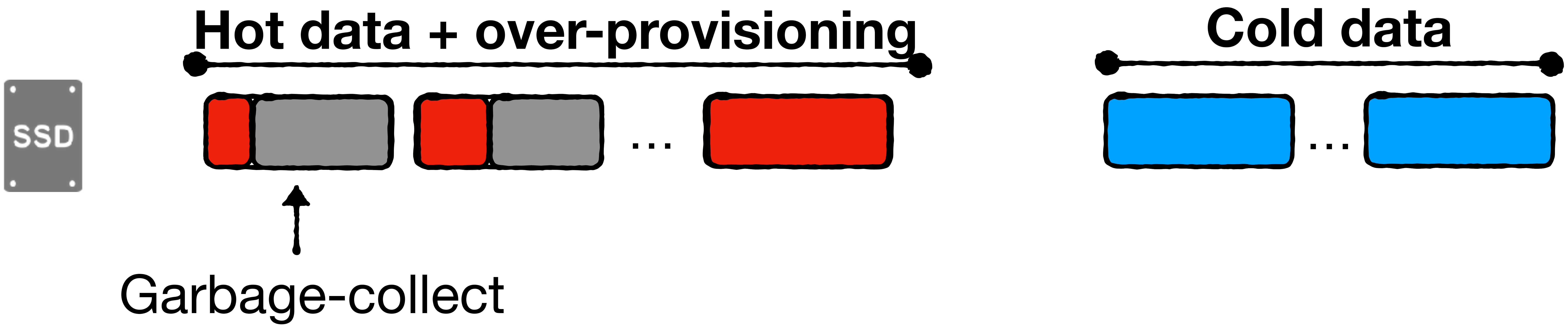
Consider a circular log where the physical capacity consists of 70% static data (never updated), 10% hot data, and 20% over-provisioning.

(A) Estimate a lower bound and upper bound for write-amplification assuming no hot/cold data separation.

(B) Estimate write-amplification assuming perfect hot/cold data separation.

Let C=0.7, H=0.1 and O=0.2

For hot/cold separation estimation, assume all over-provisioned space is applied on hot areas.



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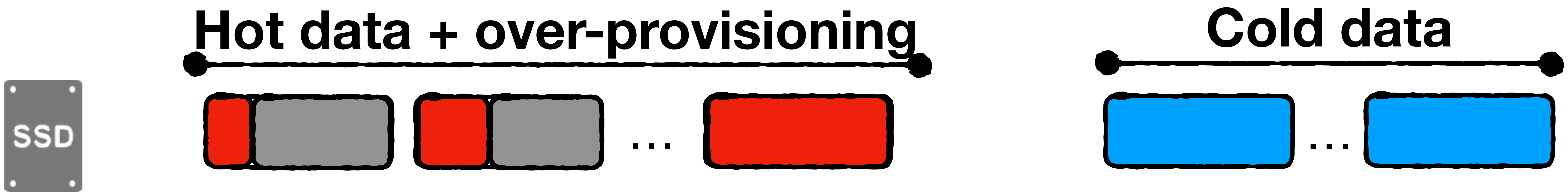

Estimation: $= 1 + \frac{1}{2} \cdot \frac{L}{O} = 1 + \frac{1}{2} \cdot \frac{H}{O} = 1 + \frac{1}{2} \cdot \frac{0.1}{0.2} = 1.25$